The Layer Laboratory: A Calculus for Additive and Subtractive Composition of Anisotropic Surface Reflectance

Supplemental Material - Integrating the layer model into a rendering system

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1 MODEL EVALUATION AND IMPORTANCE SAMPLING

In this document, we outline the necessary steps to use our layered material models in a standard Monte Carlo based rendering system. Typically, there are two interfaces to be implemented: evaluating the BSDF and importance sampling outgoing directions proportional to some prescribed probability density function (PDF). As our model is essentially a tabulated BSDF representation it admits a perfect importance sampling scheme. We propose a variation to the sampling scheme of Jakob et al. [2014] and address both the change to complex exponentials, as well as the additional dependence on the ϕ_s azimuth parameter.

1.1 Representation

We assume that we have for all discretized pairs (μ_i, μ_o) a suitable Fourier expansion available as a two-dimensional array $a_{s,d}$. Note how our BSDF representation includes the cosine foreshortening factor, allowing us to directly importance sample their product.

$$f(\mu_i, \mu_o, \phi_s, \phi_d)|\mu_o| = \sum_{s,d\in\mathbb{Z}} a_{s,d}(\mu_i, \mu_o)\; e^{i\,d\,\phi_d}\; e^{i\,s\,\phi_s} \tag{1}$$

By rearranging and renaming the same coefficients, we can also reinterpret it as an alternative Fourier expansion in ϕ_i and ϕ_d. This will be useful for importance sampling.[1]:

$$\begin{aligned} f(\mu_i, \phi_i, \mu_o, \phi_o)|\mu_o| &= \sum_{s,d\in\mathbb{Z}} a_{s,d}(\mu_i, \mu_o)\; e^{2i\,s\,\phi_i}\; e^{i\,(s+d)\,\phi_d} \\ &= \sum_{s,d\in\mathbb{Z}} b_{s,d}(\mu_i, \mu_o)\; e^{i\,s\,\phi_i} e^{i\,d\,\phi_d} \end{aligned} \tag{2}$$

For a small example with finite size $m_s = 1$ and $m_d = 2$, the coefficient tables $a_{s,d}$ and $b_{s,d}$ are arranged as follows:

$$a_{s,d}: \quad \phi_s \left\{ \underbrace{\begin{array}{|c|c|c|c|c|}\hline a_{-1,-2} & a_{-1,-1} & a_{-1,0} & a_{-1,1} & a_{-1,2} \\ \hline a_{0,-2} & a_{0,-1} & a_{0,0} & a_{0,1} & a_{0,2} \\ \hline a_{1,-2} & a_{1,-1} & a_{1,0} & a_{1,1} & a_{1,2} \\ \hline \end{array}}_{\phi_d} \right.$$

$$b_{s,d}: \quad \phi_i \left\{ \underbrace{\begin{array}{|c|c|c|c|c|c|c|}\hline a_{-1,-2} & a_{-1,-1} & a_{-1,0} & a_{-1,1} & a_{-1,2} & & \\ \hline & & & & & & \\ \hline & a_{0,-2} & a_{0,-1} & a_{0,0} & a_{0,1} & a_{0,2} & \\ \hline & & & & & & \\ \hline & & a_{1,-2} & a_{1,-1} & a_{1,0} & a_{1,1} & a_{1,2} \\ \hline \end{array}}_{\phi_d} \right.$$

1.2 Evaluation

To evaluate $f(\mu_i, \phi_i, \mu_o, \phi_o)|\mu_o|$ we distinguish two cases: For pair (μ_i, μ_o) that are part of the discretization set, we can directly evaluate the stored Fourier expansion as shown in (1) using $\phi_s := \phi_o + \phi_i, \phi_d := \phi_o - \phi_i$. Otherwise, we use the neighboring four samples along μ_i and μ_o with Catmull-Rom splines to interpolate a suitable two-dimensional coefficient array.

1.3 Importance Sampling

We sample zenith and azimuth angles in two consecutive steps to generate outgoing directions proportional to $f \cdot |\mu_o|$.

1.3.1 Sampling Outgoing Zenith Angle. Based on a given incident direction $\omega_i = (\mu_i, \phi_i)$, we first generate an outgoing angle cosine μ_o. For this, consider the one-dimensional Fourier expansion given by the values $b_{s,0}$ (2) that describe the BSDF without the dependence on the yet unkown ϕ_d:

$$b_{s,0}(\mu_i, \mu_o) = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f(\mu_i, \mu_o, \phi_i, \phi_o)|\mu_o|\, e^{-i\,s\,\phi_i}\, \mathrm{d}\phi_i\, \mathrm{d}\phi_d$$

If we interpolate these coefficients for the given μ_i and evaluate the series at ϕ_i, we get a set of discretized samples of a function in μ_o. Analogously, we get corresponding *cumulative* samples (essentially a CDF) using the (precomputed) lookup table of values

$$B_s^{(i,o)} = \int_{-1}^{\mu_o} b_{s,0}(\mu_i, \mu_o')\, \mathrm{d}\mu_o' \quad \text{where} \quad 0 \le i, o < n.$$

We use this CDF to sample a segment of this function and rely on a standard numerical inversion method based on Newton's Method, to sample an intermediate value μ_o on a spline between the two segment endpoints.

1.3.2 Sampling Outgoing Azimuth Angle. Once μ_i, μ_o, and ϕ_i are all known, we can construct a 1D Fourier expansion in ϕ_d, and sample from it:

$$f(\phi_d) = \sum_{d\in\mathbb{Z}} c_d(\mu_i, \phi_i, \mu_o)\; e^{i\,d\,\phi_d} \tag{3}$$
$$\text{where} \quad c_d(\mu_i, \phi_i, \mu_o) = \sum_{s\in\mathbb{Z}} b_{s,d}(\mu_i, \mu_o)\; e^{i\,s\,\phi_i}$$

[1] This representation is π-periodic in ϕ_i. We can still interpret it as 2π-periodic however and have zero rows in the Fourier coefficient table that we can skip during evaluation.

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We again sample by numerical inversion of its CDF:

$$\begin{aligned} F(\phi_d) &= \int_0^{\phi_d} f(\phi_d')\,\mathrm{d}\phi_d' \\ &= \int_0^{\phi_d} c_0\,\mathrm{d}\phi_d' + \sum_{\substack{d\in\mathbb{Z}\\ d\neq 0}} c_d \int_0^{\phi_d} e^{i\,d\,\phi_d'}\,\mathrm{d}\phi_d' \\ &= c_0\phi_d + \sum_{\substack{d\in\mathbb{Z}\\ d\neq 0}} \frac{i\,c_d}{d}\left(1 - e^{i\,d\,\phi_d}\right) \end{aligned} \tag{4}$$

1.4 BSDF Probability Density function

For many rendering algorithms (such as path tracing with multiple importance sampling) you also need to be able to evaluate the PDF corresponding to the sampling scheme. As outlined above it is proportional to $f(\mu_i, \phi_i, \mu_o, \phi_o)|\mu_o|$, but we have to find proper normalization constants $\rho(\mu_i, \phi_i)$ to turn this expression into a proper PDF, i.e. the following integral should equal one:

$$\int_0^{2\pi}\int_{-1}^{1} \frac{1}{\rho(\mu_i, \phi_i)} f(\mu_i, \mu_o, \phi_i, \phi_d)\,|\mu_o|\,\mathrm{d}\mu_o\,\mathrm{d}\phi_d = 1 \tag{5}$$

$$\begin{aligned} \rho(\mu_i, \phi_i) &= \int_0^{2\pi}\int_{-1}^{1} f(\mu_i, \mu_o, \phi_i, \phi_d)\,|\mu_o|\,\mathrm{d}\mu_o\,\mathrm{d}\phi_d \\ &= \int_0^{2\pi}\int_{-1}^{1}\left[\sum_{s\in\mathbb{Z}}\left(\frac{1}{2\pi}\int_0^{2\pi} f(\ldots)\,|\mu_o|\,e^{-i\,s\,\phi_i}\,\mathrm{d}\phi_i\right) e^{i\,s\,\phi_i}\right]\mathrm{d}\mu_o\,\mathrm{d}\phi_d \\ &= 2\pi\sum_{s\in\mathbb{Z}}\left[\int_{-1}^{1}\left(\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} f(\ldots)\,|\mu_o|\,e^{-i\,s\,\phi_i}\,\mathrm{d}\phi_i\,\mathrm{d}\phi_d\right)\mathrm{d}\mu_o\right] e^{i\,s\,\phi_i} \\ &= 2\pi\sum_{s\in\mathbb{Z}}\left[\int_{-1}^{1} b_{s,0}(\mu_i, \mu_o)\,\mathrm{d}\mu_o\right] e^{i\,s\,\phi_i} \\ &= 2\pi\sum_{s\in\mathbb{Z}} B_s^{(i,n-1)} e^{i\,s\,\phi_i} \end{aligned}$$

Assuming that the pair (μ_i, μ_o) is part of the set of discretized zenith angles, the values $B_s^{(i,n-1)}$ are already available in the CDF lookup table we used earlier. Otherwise we can again fall back on interpolation, like described for the BSDF evaluation above.

REFERENCES

Wenzel Jakob, Eugene d'Eon, Otto Jakob, and Steve Marschner. 2014. A Comprehensive Framework for Rendering Layered Materials. *ACM Trans. Graph.* 33, 4, Article 118 (July 2014).